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Liquidity Creation, Efficiency, and Free Banking*

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I explore the functioning of inside money competition in an overlapping generations model to address the question of whether the base of currency supply should be a monopoly. In such an economy, banks enhance allocative efficiency by offering short-term contracts (banknotes) in order to finance long-term investments since wealth has to be transferred between the generations prior to the fruition of the high-yielding long-term investments. Liquidity is created by offering agents favorable short-term contracts for funds that are earmarked for long-term investments. I study how issuance of banknotes, liquidity creation, and payment processes interact in competitive and monopolistic banking industries. I show that neither free banking (banking with free entry) nor monopoly banking achieves a first-best (Pareto-efficient) allocation. However, free banking is Pareto-inefficient compared to monopoly banking. This inefficiency arises from the overissuance incentives of competing banks. Additional liquidity needs are distributed among all banks through the payment system provided that households are indifferent to which banknotes they use to satisfy their liquidity needs. Hence, the issuer of banknotes creates a negative externality since other banks must invest more in short-term investments in order to balance liquidity needs. Under free banking, the first holders of banknotes receive an implicit return, so they can profit from funds earmarked for long-term investments. The monopoly does not provide implicit returns, but the first holders get returns from their banks' shares. Monopoly banking Pareto-dominates free banking since it has to devote a smaller portion of resources to less profitable short-term investments. *Journal of Economic Literature* Classification Numbers: E42, E50, E51, G21. © 1998 Academic Press

1. INTRODUCTION

I examine the functioning of inside money competition in an overlapping generations model. In such an economy with sequentially incomplete markets, banks may enhance allocative efficiency by offering short-term con-

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tracts (banknotes) and financing long-term investments since wealth has to be transferred between the generations prior to fruition of the high-yielding long-term investments. My analysis sheds light on the following issues.

First, with the rising relevance of modern payment instruments the issue of competing inside moneys has received renewed interest. Two of the key financial intermediation functions served by banks are liquidity creation and payment services (e.g., Diamond and Dybvig, 1983; Hellwig, 1994). The usual approach is to treat these functions separately (e.g., Freixas and Rochet, 1997). The first question I address is: how do liquidity creation and payment processes interact in competitive and monopolistic banking industries?

Second, several authors favor the outright abolition of government regulation in the banking sector, e.g., the ban on the issuance of private banknotes and entry barriers, as long as money is backed by real goods (Hayek, 1976; Klein, 1974; Vaubel, 1985). This raises the second question: should the base of currency supply be a monopoly?[1]

I explore competition among inside moneys in the form of short-term contracts backed by real goods.[2] These short-term contracts are called banknotes. I use a simple overlapping generations model with technology features of production, designed to distinguish between short-term and more profitable long-term investments. Firms carry out the long-term and short-term investments. Short- and long-term investments are financed by the issuance of banknotes through private banks. The banknotes are backed by a consumption good. The first generation has an endowment and owns banks and the firm. Shares of banks and of the firm are traded on date 2 from the first to the second generation in exchange for the consumption good.

In the model, banks perform two functions. First, by indirect financing, banks carry out the traditional intermediary function (see, e.g., the survey of Bhattacharya and Thakor, 1993, or Hellwig, 1994). Second, the first generation can profit from funds earmarked for long-term investments by trading in discounted banknotes. Hence, the first generation can participate in the efficiency gains of long-term investments. Short-term liquidity needs force the banks to engage in short-term, low-yield investments.

Free banking under a commodity standard is characterized by free entry and the unrestricted issuance of banknotes. The banknotes, however, have

[1] Several authors have conducted detailed studies of the free banking experiences, for instance, Rockoff (1974) and Rolnick and Weber (1983, 1984) for the United States, White (1984) for Scotland, and Nedwed (1992) for Switzerland. Informal discussions can be found, for instance, in Selgin (1988) and White (1984). A recent survey of the main issues is contained in Freixas and Rochet (1997).

[2] For examinations of commodity money see, e.g., Barro (1979) or Sargent and Wallace (1983).

to be backed by consumption goods so that the holder has the right to obtain a promised quantity of the consumption good upon demand. In the monopoly case, only one firm is allowed to supply banknotes.[3]

My main results are as follows. The interactions of liquidity creation and payments processes are such that neither free banking nor monopoly achieves a first-best allocation. However, monopoly banking Pareto-dominates free banking. Thus, the base of the currency supply should be a monopoly under a commodity standard.

This relative inefficiency of free banking arises from the overissuance incentives of banks in competition. Additional liquidity needs are distributed among all banks through the payment system provided that households are indifferent to which banknotes they use to satisfy their liquidity needs. Hence, the issuer creates a negative externality since other banks have to invest more in short-term investments in order to balance the liquidity needs, which leads to lower profits.

Because of the overissuance incentive, banks offer discounts on banknotes to the first holder.

This first holder receives an implicit return on holding banknotes and can profit from funds that are earmarked for longer-term investments.[4] The competitive banking industry creates more liquidity. However, to satisfy additional liquidity requirements, the banking system must devote a higher portion of real resources to short-term investments than in the monopoly case. A monopoly bank does not offer a discount to the first holder and hence accumulates profits. Therefore, the first holder receives no returns from holding money, but receives returns from trading the monopoly bank shares with the next generation. The returns from bank shares act as a substitute for the implicit returns on banknotes in the competitive case. However, the monopoly can devote a smaller portion of resources to short-term, low-yield investments. Consequently, monopoly banking Pareto-dominates free banking. This can justify a monopoly under a commodity standard being the base of the currency supply.

The paper is organized as follows. In the next section, I describe the model. In Section 3, the Arrow–Debreu solution for the model is calculated. In Section 4, I analyze the behavior of households and banks. In Section 5, the Nash equilibrium of the banknote issuance, which includes the externalities described above, is derived. In Section 6, the properties of the overall equilibrium are presented, which allows me to compare free banking and monopoly. Section 7 concludes.

[3] As long as the monopolist pursues profit maximization, it is irrelevant whether the bank is privately or publicly owned.

[4] The implicit returns stem from the discounts on issued banknotes. Discounts were a frequent phenomenon during free banking areas (e.g., Gersbach and Nedwed, 1994; Rolnick and Weber, 1983).

2. THE MODEL

In this section, I describe the model. There exists one physical good in the economy which can be used for production or consumption. The model includes three periods or three points in time. Periods are designated as $t = 1, 2, 3$. One period is equivalent to one point in time.

2.1. *Production Technology*

There exist two intertemporal production technologies. One, called storage, converts one unit of time t good into one unit of time $t + 1$ good. A second one, called production, converts one unit of time $t = 1$ good into one unit of time $t = 3$ good. The storage technology simply reproduces the physical good in the next period. The production technology yields a total return per unit invested $1 + r$, $r > 0$.

2.2. *Households*

The population consists of two overlapping generations (first and second generation) of equal size, each living for two periods. Agents have additively separable utility functions defined over consumption in the two periods:

$$u(C_t^1, C_t^2) = v(C_t^1) + \delta v(C_t^2). \tag{1}$$

$C_t^1(C_t^2)$ represents the consumption of the generation born in period t ($t = 1, 2$) when agents are young (old) and δ denotes the discount factor. I assume:

$$v''(C) < 0, \lim_{C \to 0} v'(C) = \infty. \tag{2}$$

Suppose that the number of agents is equal in each generation. This assumption is convenient, but not essential for my arguments. It allows me to analyze free banking by examining the behavior of the representative household in each period. I also follow the standard assumption in the overlapping generations literature that the substitution effect dominates the income effect; i.e., savings are an increasing function of the real interest rate.

2.3. *Endowment*

Each generation receives an endowment E of the good when young, and none at any other time. The households' objective is to smooth consumption over their lifetimes.

2.4. *Firms and Banks*

I assume that a firm f exists which carries out the long-term production process. Moreover, there are n banks indexed by $i = 1, 2, \dots, n$. Banks are assumed to be able to carry out the storage by themselves.[5] The firm and the banks live over the lifetime of the economy. They are owned by the first generation in period $t = 1$.

2.5. *Institutional Setup*

I next specify the institutional setup. For that purpose, I briefly summarize the main distinctions among various kinds of free banking.

First, competition among outside moneys is characterized by the lack of a legal claim of the bearer of outside money against the issuer. As discussed in Friedman (1960) or Hellwig (1985), a banking structure with more than one issuer would not lead to a finite price level.

Second, competition among inside moneys can occur in two different ways. On the one hand, the moneyholder may have a legal claim to obtain a specified quantity of a real good which was historically often given by the promise to exchange inside money into gold coins. On the other hand, the legal claim may represent a claim on a nominal asset. Typically, the nominal asset is high-powered money, supplied by the central bank, whose quantity cannot be directly influenced by private banks.

In the light of the distinction between the various kinds of currency competition in this paper, free banking is understood as a set of government regulations specifying only the nature of money, but no restriction of market entry. I study inside money competition, where banknotes represent the claim of the holder to obtain, upon demand, a certain specific quantity of a physical good. Banknotes do not bear any explicit interest rates. For simplicity, I assume that the parity between the unit of account and one unit of the physical good is one, i.e., one dollar represents the legal claim for one physical unit of the good in every period.

2.6. *Equilibrium Concept*

I apply two equilibrium concepts. First, I determine free banking under quantity competition, i.e., banks take the price of banknotes as given and compete on issuing a certain amount of banknotes. Second, with free entry, I also find the equilibrium under Bertrand price competition.

[5] This assumption simplifies the notation, but is not essential. Allowing households or the firm to carry out the storage yields the same results.

2.7. *Markets*

I study the following market structure and behavior of agents.

Period 1. Issuance of banknotes. Bank i issues banknotes in the amount d_i to the first generation. I denote by p the price of these banknotes in terms of dollars. Accordingly, $1 - p$ represents the discount that banks are willing to give to their clients during issuance.[6] I assume $0 \leq p \leq 1$. Discounts during issuing periods were a frequent phenomenon in free banking areas (e.g., Gersbach and Nedwed, 1994; Rolnick and Weber, 1983). Bank i receives pd_i time-1 goods in the first period.

Credit market. Bank i devotes a certain part, denoted by R_i, of the received time-1 goods to its own storage as liquidity reserves. The rest, denoted by K_i, is supplied as credits to firm f for investments. The creditor–debtor relation is represented by a standard credit contract with fixed repayment of the principal and of the interest in the third period. Total reserve holdings and credits of the banking industry are denoted by R and K, respectively.

Period 2. Good and equity market. The first generation offers the banknotes to the next generation in exchange for time-2 goods. Similarly, the shares of the firm and of the banks are supplied to the next generation in the equity market in exchange for time-2 goods.

Reflux of banknotes and outflow of reserves. All remaining banknotes that are still in the hands of the first generation are presented to the banks which have to fulfill their obligations by the exchange of one unit of time-2 goods for one dollar.

Period 3. Credit market. Firm f pays back the principal and the contracted interest payment to the banks in time-3 goods.

Reflux of banknotes and outflow of reserves. The second generation exchanges its banknotes at the banks for time-3 goods.

Dividend payments. The firm and the banks are liquidated and all remaining real goods are distributed as dividends among the second generation. The dividend payments, which are equal to the profits evaluated in the third period, are denoted by G_f and G_i, respectively. The sum of the banks' profits is denoted by G.

2.8. *Remarks on Frictions*

My model involves two frictions in the functioning of markets. First, I follow the literature in assuming that firms cannot finance their investments by issuing equities. A variety of reasons can be found in the literature

[6] Of course, the discounts must be limited to the issuance period when accounts are opened since otherwise arbitrage would lead to bankruptcy of the banks. Note also that the public usually has the right to get an arbitrary amount of banknotes at the parity one.

to justify this assumption, such as nonconvexities and monitoring (e.g., Bhattacharya and Thakor, 1993). For example, if banks are able to perfectly monitor what firms are doing with their funds, banks fulfill their intermediary function by monitoring that firms actually invest and do not divert funds. I do not explicitly model this friction. The second friction in my model is the incompleteness (or incomplete participation) of markets since generations overlap. Hence, there is space to enhance allocative efficiency by offering suitable short-term contracts and financing long-term contracts with higher returns.

In summary, banks fulfill two interrelated functions in my economy. On the one hand, by indirect financing, banks carry out the traditional intermediary function. On the other hand, the first generation can profit from funds that it earmarked for long-term investments because of the discounts on issued banknotes. This liquidity provision is different from that discussed in the literature. There, banks are able to provide liquidity by exploiting the law of large numbers in a risky environment (see, e.g., Hellwig 1994). In my case, the liquidity provision stems from the fact that the money issuer gives the first money holder some implicit interest payments from longer-term investments. The money issuer can invest correspondingly in long-term investments in order to pay the interest, whereas the first generation cannot (see also Diamond and Dybvig 1983). In my model, there is no uncertainty. I can thus ignore all difficulties arising from asymmetric information.[7]

3. FIRST-BEST

I first establish the properties of a Pareto efficient allocation, i.e., the first-best. The first-best outcome represents the benchmark, based on the Arrow–Debreu version of the above model.

In an Arrow–Debreu world, all goods markets are simultaneously open in the first period. Thus, the first household supplies its endowments and demands time-1 goods for consumption and storage and time-2 goods for consumption. Similarly, the second generation supplies its endowments and demands time-2 and time-3 goods. Finally, the first generation owns the firm and receives dividends in terms of time-3 goods that can be sold in the market for time-3 goods.

I denote by $q = (q_1, q_2, q_3)$ the vector of commodity prices, by $K(q_1, q_3)$ the investment demand of firm f, and by $S_f^3(q_1, q_3)$ the supply of time-3 good by firm $f \cdot q_t$ is the price of time t good in \$. D_t^1 (D_t^2) represents the

[7] Information asymmetries between banks, depositors, and creditors are another reason why people argue against free banking (e.g., Chu, 1993).

demand of the generation born in period t ($t = 1, 2$) when agents are young (old). L_t represents the amount of resources devoted to storage by the generation born in period t ($t = 1, 2$). Consumption, demand, and storage are assumed to be nonnegative. Since agents are never satiated, consumption, demand, and storage satisfy the following relationships:

$$D_t^1 = C_t^1 + L_t \quad \text{and} \quad D_t^2 = C_t^2 - L_t. \tag{3}$$

Finally, I assume that the firm is owned by the first generation. π_f denotes the real value of dividends in terms of time-3 goods. Since all markets are open simultaneously, the first generation does not need to trade the shares of the firm, since it can sell the real value of the dividends in the market for time-3 goods.

I examine the behavior of the firm f. The nominal profits are $q_3\pi_f$ and given by

$$q_3\pi_f = K(q_1, q_3)(1 + r)q_3 - K(q_1, q_3)q_1 = K(q_1, q_3)\{(1 + r)q_3 - q_1\}. \tag{4}$$

Since the profit function is linear in $K(q_1, q_3)$, I immediately obtain:

$$K(q_1, q_3) = \begin{cases} \infty & \text{if } q_1 < q_3(1 + r) \\ \text{arbitrary} & \text{if } q_1 = q_3(1 + r) \\ 0 & \text{if } q_1 > q_3(1 + r). \end{cases} \tag{5}$$

Moreover, I have

$$S_f^3(q_1, q_3) = (1 + r)K(q_1, q_3). \tag{6}$$

Households maximize utility subject to their budget constraint. I consider the first generation:

$$\begin{aligned} &\text{Max}\{u(C_1^1, C_1^2) = v(C_1^1) + \delta v(C_1^2)\} \\ &\text{s.t. } q_1 D_1^1 + q_2 D_1^2 = q_1 E + q_3 \pi_f. \end{aligned} \tag{7}$$

Using the relationships embodied in (3), the budget constraint can be written as

$$q_1 C_1^1 + q_2 C_1^2 = q_1 E + (q_2 - q_1)L_1 + q_3\pi_f. \tag{8}$$

Similarly, the problem of the second generation can be written

$$\begin{aligned} &\text{Max}\{u(C_2^1, C_2^2) = v(C_2^1) + \delta v(C_2^2)\} \\ &\text{s.t. } q_2 C_2^1 + q_3 C_2^2 = q_2 E + (q_3 - q_2)L_2. \end{aligned} \tag{9}$$

Before I proceed with the characterization of the first-best allocation, arbitrage arguments allow me to simplify the analysis:

LEMMA. *Every Arrow–Debreu equilibrium satisfies:*

$$\text{(i)} \quad q_1 \geq q_2 \geq q_3$$

$$\text{(ii)} \quad q_3 = \frac{q_1}{1+r}.$$

Proof. The first part follows from the maximization problem of the households. Suppose, for example, $q_1 < q_2$. The first generation has the technology to convert time-1 goods into time-2 goods by storing the goods. The agents demand and supply goods depending on the relative prices in the economy. They do not calculate with overall resource constraints in the economy. Thus, for $q_1 < q_2$, the first generation would demand an infinite number of time-1 goods in order to sell them as time-2 goods. Thus:

$$L_1(q_1, q_2) = \begin{cases} \infty & \text{if } q_1 < q_2 \\ \text{arbitrary} & \text{if } q_1 = q_2 \\ 0 & \text{if } q_1 > q_2. \end{cases} \tag{10}$$

The first case, $q_1 < q_2$, is not feasible, due to resource constraints. Thus, in any equilibrium, I have $q_1 \geq q_2$. The same argument establishes that $q_2 \geq q_3$.

The second part results from the profit maximization of the firm. Suppose that $q_3 > q_1/(1+r)$. Then investment demand will be infinite, which cannot be an equilibrium because of resource constraints. Suppose that $q_3 < q_1/(1+r)$. Then investment demand as well as the supply of time-3 goods would be zero. By my assumptions (Eq. (2)) and since any equilibrium requires that $q_1 \geq q_2 \geq q_3$, each generation demands a nonnegative amount of consumption in every period. As the investment demand and the supply of time-3 goods are zero, any equilibrium would require that L_1 and L_2 are strictly positive. From the first part of the lemma, I know that storage of both generations can only be positive if $q_1 = q_2 = q_3$. This, however, contradicts $q_1 > q_3(1+r)$. Hence, in any equilibrium I must have $q_1 = q_3(1+r)$ and thus profits of the firm are zero. Q.E.D.

Using the lemma, I restrict my analysis to price vectors $q_1 \geq q_2 \geq q_3 = q_1/(1+r)$. In particular, I can ignore the possibility of arbitrage by storage. Moreover, the first generation calculates its demand for goods anticipating that dividends of the firm will be zero.

Hence, for the restricted domain $q_1 \geq q_2 \geq q_3 = q_1/(1 + r)$, $\pi_f = 0$, I obtain the standard first-order conditions

$$\frac{\partial v(C_t^1)}{\partial C_t^1} \bigg/ \left\{ \delta \frac{\partial v(C_t^2)}{\partial C_t^2} \right\} = \frac{q_t}{q_{t+1}}. \tag{11}$$

Let me introduce short-term interest rates: $r_1 = q_1/q_2 - 1$, $r_2 = q_2/q_3 - 1$. Note that these interest rates are on commodities. Clearly, the utility of a generation increases if the corresponding short-term interest rate increases, since r_1 and r_2 determine the returns on savings of each generation. I proceed now as follows. I first characterize the overall equilibrium in terms of prices. Then I interpret the equilibrium in terms of interest rates and utilities.

An equilibrium set of prices must fulfill the equilibrium conditions in the goods market in every period:

$$\text{First period: } K + C_1^1 + L_1 = E \tag{12}$$

$$\text{Second period: } C_1^2 + C_2^1 + L_2 = E + L_1 \tag{13}$$

$$\text{Third period: } C_2^2 = L_2 + (1 + r)K. \tag{14}$$

Using the condition $q_1 \geq q_2 \geq q_3$ imposed by the lemma on the set of equilibrium prices, I examine the following three cases:

$$\text{(a)} \quad q_1 = q_2 > q_3 = \frac{q_1}{1 + r}$$

$$\text{(b)} \quad q_1 > q_2 = q_3 = \frac{q_1}{1 + r}$$

$$\text{(c)} \quad q_1 > q_2 > q_3 = \frac{q_1}{1 + r}.$$

In the next proposition, the equilibrium can be characterized according to the above cases:

PROPOSITION 1. *Suppose that $v(\cdot)$ is strictly concave and that the substitution effect dominates the income effect. Then a unique equilibrium exists, characterized by*

$$q_1 > q_2 > q_3 = \frac{q_1}{1 + r}. \tag{15}$$

The proof is given in the Appendix. I also interpret the equilibrium in terms of interest rates and utilities. Clearly,

$$(1 + r_1)(1 + r_2) = 1 + r. \tag{16}$$

In equilibrium, the long-term interest rate is compounded of two short-term interest rates, one for each generation.

The following economic problem needs to be solved. On the one hand, the first generation should be induced to save in order to generate gains from long-term investments. On the other hand, the first generation would like to participate in the investment gains. Since the substitution effect dominates the income effect, the first generation saves more if the short-term interest rate is higher. This is compatible to some extent with higher investments in long-term capital and therefore higher participation of the first generation in the investment gains.

4. THE BEHAVIOR OF HOUSEHOLDS AND BANKS

4.1. *Preliminary Remarks on Equilibrium*

Before I investigate the behavior of households and banks and the resulting equilibrium allocation, I observe that the price in the goods market in period two must be 1. Assume that the first household acquires some banknotes in the first period. If the price in the second period was higher than 1, the second generation would not be willing to give away any of its resources, since it could store its own endowment. The first generation, however, offers all banknotes to the next generation. If the price were lower than 1, the first generation would not offer any banknotes since it would be more profitable to bring them back to the banks in order to convert them into time-2 goods at the parity. The second generation, however, would be happy to obtain some banknotes.

The above considerations imply that the interest rate between periods $t = 2$ and $t = 3$ is zero. This also means, by an arbitrage argument, that in equilibrium of the equity market, the prices of the shares in the second period must be equal to the dividends received in the last period.

Because of the assumption that the long-term technology is linear, an equilibrium in the credit market requires that r be the equilibrium interest rate. Otherwise, firms would either be willing to invest infinite resources or not be interested in investing anything at all. The equilibrium interest rate r also implies that the firm earns zero profits and, thus, cannot distribute any dividends in the last period ($G_f = 0$).

4.2. *Households*

I now examine the behavior of households while simplifying the analysis in two respects. First, in the derivation of consumption plans, I neglect the trade of the ownership of the firm, because I have $G_f = 0$ in equilibrium and thus the prices of firm's shares are zero. Second, although the second generation can buy shares of banks with its endowment, this cannot enhance the saving possibilities of the second generation since buying bank shares and acquiring banknotes are riskless assets and thus complete substitutes. Thus, market equilibrium requires that both yield zero interest rates and the second generation is indifferent between the two assets. The amounts of bank shares and banknotes that are acquired by the second generation are determined by market equilibrium considerations, discussed in the next section.

The interest rate between periods $t = 2$ and $t = 3$ is zero since the second generation does not receive a discount. Hence, the second generation solves the following problem:

$$\max\{u(C_2^1, C_2^2) = v(C_2^1) + \delta v(C_2^2)\} \tag{17}$$

$$\text{s.t. } C_2^1 + C_2^2 = E. \tag{18}$$

I denote these solutions of the second generation's optimization by C_2^1 and C_2^2. The first generation solves the following problem:

$$\max\{u(C_1^1, C_1^2) = v(C_1^1) + \delta v(C_1^2)\} \tag{19}$$

$$\text{s.t. } p^{-1}C_1^1 + C_1^2 = p^{-1}E + B. \tag{20}$$

p is the price of banknotes. B denotes the value of all bank shares in units of time-2 goods when they are traded in the stock market in the second period. Note that the first generation owns the banks and the firm and thus receives additional income in the second period when they can sell the bank shares.

As discussed above, the gross return on holding bank shares for the second generation is 1. Therefore, B equals the dividends from holding bank shares in terms of time-3 goods. In other words, the value of the shares is equal to the sum of banks' profits. Since B equals G, I replace B by G in the optimization problem of the household. Denote the solutions of the problem of the first generation by $C_1^1(p, G)$ and $C_1^2(p, G)$. Note that the first generation receives 1 unit of the goods from the banks for \$1.00 in the second period, whereas in the first period it gets \$ $(1/p)$ for 1 unit of the goods supplied to the bank.

4.3. *Banks*

I consider the behavior of the banks. I first derive the profits for an individual bank i:

$$G_i = (r+1)K_i - K_i + d_i p - d_i \\ = rK_i + d_i(p-1). \tag{21}$$

An individual bank i has the trade-off between losses arising from discounting and profits from investments in long-term production. The resources allocated to credits K_i are determined by the difference between the received time-1 goods $d_i p$ and the necessary reserve holdings R_i for the second period. Thus, using $K_i = d_i p - R_i$, I obtain

$$G_i = d_i(p-1) + r(d_i p - R_i). \tag{22}$$

Next, I calculate the reserve holdings, i.e., the amount of real resources banks devote to storage. After deriving the total reserve holding of the banking system, I can calculate individual reserve holdings. The second generation is willing to acquire $E - C_2^1 - G$ banknotes after they have bought the shares of the banks.[8] Thus, total reserve holdings of the banking system for the second period amount to[9]

$$R = \Sigma\, d_j - (E - C_2^1 - G). \tag{23}$$

Given total reserve holdings of the banking system, I can specify individual reserve holdings. The first generation is indifferent regarding which banknote, and from which bank, it offers to the next generation, and which banknote it presents to the banks. Thus, a natural way of defining individual reserve holdings is to assume that

$$R_i = \frac{d_i}{\Sigma\, d_j}\{\Sigma\, d_j - (E - C_2^1 - G)\}. \tag{24}$$

This assumption is based on the view that each banknote has an equal probability of being presented to the banks. There are no preferences of the public concerning the withdrawal of reserves from certain banks. Thus, I assume that a bank expects reserve losses as an amount of the reserve

[8] One could also simultaneously model all markets in the second period. This would lead to the same equilibrium. The sequential analysis clarifies the resulting reflux of banknotes and the outflow of reserves.

[9] It will be shown that reserve holdings can never be negative.

losses of the whole banking system proportional to its market share. The bigger the bank, the higher the expected reserve holdings are, since the probability of the public presenting banknotes in exchange for the promised goods is greater (see also Gersbach and Nedwed, 1994). In order to work without uncertainty considerations, I apply the law of large numbers.[10]

Note that I do not need to assume that banknotes are indistinguishable to the public. Banknotes can have bank-specific pictures as long as the banknotes are viewed as perfect substitutes in their use. As was the case in historical free banking episodes, banknotes had bank-specific pictures and banks only honored their own moneys. Equivalent is the assumption that banks honor all moneys, but they can simultaneously exchange foreign banknotes at the origin bank. Since the probability of being presented by the first generation is the same for each banknote, the reflux of banknotes in both cases also follows Eq. (24). An individual bank expects a reflux of its own banknotes proportional to its market share, either directly from the public or indirectly through other banks.

Since G_i is dependent on R_i and R_i is itself dependent on the profits of the banking industry, I determine the solution of G_i under the assumption that banks optimally determine their reserve holdings as a function of their issuance behavior, given the behavior of other banks. Overall profits G equal $\Sigma\, d_j(p - 1) + rK$. Moreover, K can be derived as $K = p\Sigma\, d_j - R$. Inserting R from Eq. (23) yields

$$G = \Sigma\, d_j(p - 1) + r\left\{p\,\Sigma\, d_j - \left(\Sigma\, d_j - (E - C_2^1 - G)\right)\right\}. \tag{25}$$

Solving for G leads to

$$G = \Sigma\, d_j(p - 1) + \frac{r}{1 + r}(E - C_2^1). \tag{26}$$

Inserting (26) and (24) into (22) leads to the final representation of the bank's profit as follows:

$$G_i = d_i(p - 1) + \frac{d_i}{\Sigma\, d_j}\left(\frac{r}{1 + r}\right)\{E - C_2^1\}. \tag{27}$$

Equation (27) can also be obtained directly by using

[10] I could also incorporate explicitly small variances in the reflux, which, however, would need an explicit discussion of interbank markets in the second period to balance liquidity needs.

$$G_i = \frac{d_i}{\sum d_j} G.$$

Equation (27) is central for my further examinations. Note that the profit of bank i depends on the issuance behavior of other banks. If many banknotes are issued, several of them are presented to the banks in the second period in exchange for goods. This, however, forces the banks to be very cautious in their credit supply, which in turn reduces their profits. Note that in Eq. (27) only the number of banknotes issued remains as the choice variable. The number of credits is given by the following:

$$K_i = d_i p - R_i = d_i p - \frac{d_i}{\sum d_j} \left\{ \sum d_j - (E - C_2^1 - G) \right\} \quad \text{and}$$

$$K = p \sum d_j - R = p \sum d_j - \left\{ \sum d_j - (E - C_2^1 - G) \right\}$$

$$= (p - 1) \sum d_j + (E - C_2^1 - G).$$

An individual bank is allowed to break the global constraint on the amount of long-term investments and, therefore, the global reserve constraint. Hence, for the optimization problem of bank i, K_i is not restricted by K. Each bank is free to allocate capital, and it is possible that one bank plans a larger amount of credit than is feasible for the banking system. However, if an individual bank tries to issue more banknotes in order to increase long-term investments at the expense of other banks' credit volume, the bank will make losses on each newly issued banknote owing to the discounts. Since the increase in credits will become smaller due to higher reserve holdings, the gains from additional long-term investments cannot compensate the losses on discounts at a certain level of issuance. In the next section, I show that in a symmetric equilibrium the possibility of breaking the global reserve or credit constraint is not optimal for an individual bank.

5. EQUILIBRIUM OF ISSUE BEHAVIOR

In this section, I describe the features of banking behavior in equilibrium. In order to isolate the externalities arising from banknote issuance and the payment system, I focus on Walras' equilibria with externalities. Under this equilibrium notion, banks take their influence on the banknote reflux and reserve outflows into account, given the discount $1 - p$. In the limiting case, where n goes to infinity, I also obtain the Bertrand case with zero profits. In the following proposition, I characterize the supply behavior of banks, depending on the discount and the number of banks. I denote by $d(n, p)$ the banknotes issued by an individual bank in a symmetrical Nash equilibrium of banks' issue behavior. In the following, I treat the variable n as a real variable so that I can work with derivatives. I obtain the following proposition:

PROPOSITION 2. *For a given discount* $1 - p$ *and* $n > 1$, *a unique Nash equilibrium of banks' issue behavior exists:*

$$1. \quad d(n,p) = \frac{r(E - C_2^1)(n-1)}{(1+r)n^2(1-p)}.$$

For given discount $1 - p$, *the aggregate supply of banknotes* $nd(n, p)$ *increases with the number of banks:*

$$2. \quad \frac{\partial\{nd(n,p)\}}{\partial n} > 0.$$

The proof is given in the Appendix.

Proposition 2 shows that, given any discount $1 - p$, the more banks are active, the higher is the aggregate supply of banknotes. This represents the fact that the larger the number of banks that are active, the higher is the individual bank's incentive to issue banknotes since additional reserve holdings are distributed among all banks.

To complement my analysis of the issue behavior, I briefly discuss the monopoly case. In this case, if d_1 is large enough, Eq. (27) is reduced to

$$G_1 = d_1(p-1) + (E - C_2^1)\frac{r}{1+r}. \tag{28}$$

The first derivative becomes

$$\frac{\partial G_1}{\partial d_1} = p - 1 \le 0.$$

Thus, the monopoly will not offer any discount and therefore can harvest the full gains from investing in the long-term technology. We summarize the monopoly's issue behavior in the following proposition:

PROPOSITION 3. *In the monopoly case, the bank's issue behavior is characterized by*

$$d(p) = \begin{cases} \text{arbitrary} & \text{if } p = 1 \\ 0 & \text{if } p < 1. \end{cases}$$

6. TOTAL EQUILIBRIUM

After the characterization of banks' optimal supply behavior for the set of parameter values, I derive the overall equilibrium in this section. Equilibrium in the goods market in the first period requires that

$$E = C_1^1(p, G) + pnd. \tag{29}$$

Define $p(n)$, $R(n)$, and $G(n)$ as comparative static functions implied by the equilibrium equation (29) and dependent on the number of banks. I obtain:

PROPOSITION 4. *In equilibrium, I have*

1. $G(n) = \dfrac{r(E - C_2^1)}{(1 + r)n}$
2. $\dfrac{dG}{dn} < 0$
3. $\lim_{n\to\infty} G(n) = 0.$

The proof is given in the Appendix.

This proposition shows that the aggregate of banks' profits decreases when the number of banks increases. This fact rests on the externality in the payment process. A bank can increase its issuance of banknotes without bearing all additional reserve holdings since the reflux of banknotes in the second period affects all banks according to their market share. In the limit, aggregate profits vanish.

PROPOSITION 5. *For every n, there exists a unique equilibrium, characterized by:*

1. $p = 1$ for $n = 1$
2. $\dfrac{dp}{dn} < 0$

3. $K(n) = \dfrac{E - C_2^1}{1 + r}.$

The proof is given in the Appendix.

The last fact is obvious, since all time-3 goods stem from the investment in the first period. Thus, in equilibrium, the second generation just consumes $E - C_2^1$ in the third period, which otherwise requires an investment of $(E - C_2^1)/(1 + r)$. $K(n)$ is independent of n. The third point is the equilibrium result of the overissuance phenomenon of the banknotes supply.

PROPOSITION 6.

1. *Free banking is inefficient.*
2. $R(\infty) > R(1)$.

The proof is given in the Appendix.

The inefficiency of free banking is not obvious. The second generation attains the same utility level in both cases, free banking and monopoly banking. Moreover, under free banking, the first household receives some implicit returns on its banknotes, whereas the monopoly supplies only non-interest-bearing banknotes. In the monopoly case, the first generation receives a return from its bank shares that acts as a substitute to the interests on money under free banking. Moreover, under free banking, the banking system devotes a higher amount of real resources to short-term investments with zero interest rates. This essentially causes the inefficiency of free banking.

Until now I have considered quantity competition with externalities; i.e., banks compete in the issuance of deposits. In the following, I demonstrate the solution when banks compete in Bertrand fashion, i.e., they compete in discounts p.

Since reflux considerations in the second period are unchanged, Eq. (27) still represents the profit function in this case, but now banks choose individual price discounts, denoted by p_i. If bank i offers the best discount, it needs to take into account the whole reflux and hence

$$G_i = d_i(p_i - 1) + \left(\frac{r}{1 + r}\right)\{E - C_2^1\}. \tag{30}$$

The number of banknotes is determined by the demand of the first generation at discounts p_i and the profits of bank i. If bank i offers the lowest discount together with some other bank, then they share the number of banknotes they can issue. In this case, I have

$$G_i = \frac{\Sigma\, d_j}{2}\,(p_i - 1) + \frac{1}{2}\left(\frac{r}{1+r}\right)\{E - C_2^1\}. \tag{31}$$

Bertrand competition ensures that profits are zero. Thus, I obtain the equilibrium discount p^* as

$$p^* = -\frac{1}{\Sigma\, d_j}\left(\frac{r}{1+r}\right)\{E - C_2^1\} + 1. \tag{32}$$

On the other hand, note that the first generation receives time-2 goods in the same number as it has acquired banknotes. Hence:

$$\Sigma\, d_j = C_1^2(p^*, 0). \tag{33}$$

Combining both equations leads to an implicit equation for the equilibrium discount:

$$C_1^2(p^*, 0) = \frac{r(E - C_2^1)}{(1+r)(1-p^*)}. \tag{34}$$

I obtain the same discount as under free banking derived in Eq. (A1) of the proof of Proposition 6 in the Appendix. Since aggregate profits are also zero in both cases, Bertrand competition yields the same allocation as free banking; i.e., Bertrand competition is the limit case of quantity competition with free entry.

7. DISCUSSION

In my model, banks contribute to complement the market structure by creating liquidity. However, the concept of liquidity creation is different than in the bank-run literature. In the bank-run literature, following the seminal work of Bryant (1980) and Diamond and Dybvig (1983), banks contribute to complementing the market structure by allowing customers the right to withdraw their funds on demand. All customers start investing at the same time. This is feasible for banks because in the aggregate at least some of the individual uncertainty about the timing of the consumption needs of agents washes out. Maturity transformation, however, may not be stable since coordination failures among depositors are possible; this requires a careful analysis of the insurance and liquidity function of banks (see von Thadden, 1997; Hellwig, 1994; Bhattacharya and Padilla, 1996) with implications for banking regulation (see Baltensperger and Dermine, 1987). Moreover, if depositors are mobile across banks, clearing and settlement of interbank payments can become an additional source of runs which, in turn, can be alleviated by well-enforced reserve requirements (McAndrews and Roberds, 1995). In my model, only one generation can invest in long-term, high-yielding projects. The economic problem to be solved is to induce the first generation to invest, even if it is certain that the investor will never see the fruits from the investments. Uncertainty and insurance against consumption risks, as well as against corresponding withdrawal risks, are not present since only the certain withdrawal of the first generation is of importance.

My results may also shed light on the relationship between money and interest rates. A standard argument suggests that non-interest-bearing money is inefficient compared to interest-bearing money, since the private incentives for holding money are too small (Friedman, 1960; Bewely, 1979; Hellwig, 1980). I have shown that this argument needs to be adjusted when free banking and monopoly banking are compared in a model with short-term and long-term production. Interest-bearing money requires more short-term, but less profitable investments that may not be desirable.

If I compare the first-best allocation with the monopoly solution, I conclude that the monopoly case is only second-best; the second generation has too little incentive to contribute to long-term investments, since it does not obtain interest on money.

My model can be extended in several directions. I could allow interbank markets and settlements of interbank payments in the second period. Then the choice of banks' issuing behavior would become dependent on the possibility of acquiring additional reserves in the second period through interbank markets. This also introduces the possibility of strategic reserve holdings and compensating balances. That is, one bank may invest mainly in long-term investments, whereas another bank holds a large amount of reserves in order to profit from high short-term interests on interbank payments. Banks then would be engaged in intertemporal two-sided competition, which introduces a set of new strategic considerations above the traditional simultaneous two-sided competition (Stahl, 1988; Yanelle, 1989) that has been extended to a product differentiation setting by Gehrig (1996). By capturing the whole market in the first period, a bank can achieve a monopoly position in the second period. However, refinancing consideration limits this monopoly power. Since the inefficiency of free banking also occurs under Bertrand competition, it is unlikely that the inefficiency will be totally alleviated by the introduction of interbank payments and settlement systems (see Gersbach, 1997).

A further extension of my model is to examine different organizations of the credit market. Until now I have considered the credit market as perfectly competitive. Since the technology is linear and the real rate of returns on investments is r, the competitive equilibrium requires firms to be willing to invest a nonzero, finite amount of resources, which is only possible at the interest rate r. Another plausible assumption is that banks compete in prices, but have capacity constraints yielding simultaneous two-sided competition in quantity or prices (deposits) and prices (loans) and capacities in the credit market. As it has been established (see Stahl, 1988; Yanelle, 1989), simultaneous two-sided competition can lead to departures from the competitive outcome. In my model, the additional complication arises that there are not enough funds available in the market as soon as the interest rate is below r. In subsequent research I will examine whether or not the perfect elasticity of firm's demand for funds at the interest rate r will yield the competitive outcome as a result of the simultaneous two-sided competition.

Another plausible set of assumptions is a sequential procedure in which banks first obtain resources from households by offering banknotes and then bargain with firms about interest rates and quantity pairs and thus about the split of surplus. As competition among banks intensifies, the interest rate charged to the firms would decrease, since the number of firms remains constant and bargaining power shifts to firms. Since firms' profits are positive, the first generation can benefit from selling shares of firms, while the value of bank shares decreases. Both effects must be considered in the calculation of the reflux of banknotes. What happens to the overissuance incentive of banks, and finally to the inefficiency of free banking will be left for subsequent research.

Overall, the preceding discussion illustrates that the detailed organization of the credit market could be important for the comparison of monopoly and free banking.

8. CONCLUSION

In this paper, I have analyzed free banking under a commodity standard. It is not surprising that neither free banking nor monopoly banking achieves a first-best allocation. In both cases, the second generation receives no returns on money holdings. Its incentive for holding money is too weak for Pareto optimality. However, free banking is inefficient compared to a monopoly. This inefficiency of inside money competition arises from the overissuance incentives of banks in competition. If a bank issues more banknotes and invests more in long-term capital, additional liquidity needs are distributed among all banks provided that households are indifferent to which banknotes they use in exchange. This suggests that the base of the currency supply should be a monopoly.

Free banking is less efficient than monopoly despite the fact that free banking provides the first holder of the banknotes with some implicit interest. Indeed, it is the excess interest phenomenon of free banking that leads to its inefficiency, since the intertemporal allocation of investments is biased in favor of short-term low profitable investments.

To show the externalities in the liquidity provision through the payment system, I simplified the analysis in various respects. For instance, the model cannot handle inside moneys in terms of differentiated products, including additional moral hazard and bankruptcy issues. Moreover, I have focused on banknotes rather than on deposits and have neglected refinancing considerations and relationship banking (for discussions see Gersbach, 1997 and Boot, 1995).

However, I believe that the model illuminates a number of issues which are relevant not only to currency competition under a commodity standard, but also to the present currency constitution in many countries, where banks compete for the issuance of inside moneys backed by high-powered government money. In the end, the externalities and effects arising from the payment system are one of the fundamental issues in banking. This must be addressed by an analysis of currency competition.

APPENDIX

Proof of Proposition 1.

$$\text{Case b. } \left(q_1 > q_2 = q_3 = \frac{q_1}{(1+r)} \right).$$

I first rule out case b being an equilibrium. Clearly, L_1 would be zero under b. On the other hand, $C_1^2(q_1, q_2)$ is greater than $C_2^2(q_2, q_3)$, since I have assumed that $v(\cdot)$ is strictly concave and that the substitution effect dominates the income effect. Therefore, the lower relative price for household 1 for the time-2 good will induce higher consumption in the second period. The equilibrium condition in the second period can be written

$$\begin{aligned} E &= C_1^2(q_1, q_2) + C_2^1(q_2, q_3) + L_2 \\ &> C_2^2(q_2, q_3) + C_2^1(q_2, q_3) + L_2 \\ &= E + L_2. \end{aligned}$$

This would imply that L_2 must be negative, which is impossible.

I now consider a and c

$$\text{Case a. } \left(q_1 = q_2 > q_3 = \frac{q_1}{(1+r)}\right).$$

Since the substitution effect dominates the income effect, $C_1^1(q_1, q_2) > C_2^1(q_2, q_3)$. Moreover, I have $L_2 = 0$. The budget constraint of the first generation implies that

$$C_1^1(q_1, q_2) + C_1^2(q_1, q_2) = E.$$

Equilibrium in the second period requires that

$$C_1^2(q_1, q_2) + C_2^1(q_2, q_3) = E + L_1 \geq E,$$

which is impossible since $C_2^1(q_2, q_3) < C_1^1(q_1, q_2)$.

$$\text{Case c. } \left(q_1 > q_2 > q_3 = \frac{q_1}{(1+r)}\right).$$

Note that in this case $L_1 = L_2 = 0$. I assert that case c represents the unique equilibrium. To see this, I rewrite the equilibrium conditions (12), (13), (14) in the form

$$C_2^2(q_2, q_3)/(1+r) + C_1^1(q_1, q_2) = E$$
$$C_1^2(q_1, q_2) + C_2^1(q_2, q_3) = E.$$

Clearly, I can set $q_1 = 1$ and $q_3 = 1/(1 + r)$. For $q_2 = q_1$ the left side of the first expression is greater than E, whereas the left side of the second expression is smaller than E. The opposite is true for $q_2 = q_3$. On the other hand, $C_2^2(q_2, q_3)/(1 + r) + C_1^1(q_1, q_2)$ is increasing in q_2, whereas $C_1^2(q_1, q_2) + C_2^1(q_2, q_3)$ is decreasing in q_2. Thus, by the mean value theorem, a unique equilibrium value q_2 exists, since by Walras' law if one market clears, the other clears as well. This essentially completes the proof.

Q.E.D.

Proof of Proposition 2. I derive the first-order condition for bank i, given the issuance behavior of the other banks. Thus, I obtain

$$\frac{dG_i}{dd_i} = p - 1 + \frac{r}{1+r}(E - C_2^1)\left\{\frac{\sum_{j \neq i} d_j}{(\sum_j d_j)^2}\right\} = 0.$$

Clearly, the second-order condition is also fulfilled. I am looking for symmetrical Nash equilibria, which implies

$$n^2 d(1-p)(1+r) = r(E - C_2^1)(n-1).$$

Finally, I obtain

$$d(n,p) = \frac{r(E - C_2^1)(n-1)}{(1+r)n^2(1-p)}.$$

The proof of the second part immediately follows from the fact that $(n-1)/n$ is increasing with n. Q.E.D.

Proof of Proposition 4. By combining the representation of G in Eq. (26) and the derived equilibrium values of bank issue behavior $d(n, p)$, I obtain

$$G(n) = -\frac{r(E - C_2^1)(n-1)}{(1+r)n} + \frac{r}{1+r}(E - C_2^1).$$

This can be condensed to the expression represented in Proposition 3:

$$G(n) = \frac{r(E - C_2^1)}{(1+r)n}.$$

$G(n)$ is independent of p. Obviously, the higher n, the lower the banks' profits. Moreover, when n goes to infinity, G goes to zero. Q.E.D.

Proof of Proposition 5. The equilibrium condition (29) can be combined with the budget constraint of the first generation, embodied by Eq. (20). Thus, I obtain

$$C_1^2(p, G) - G = nd.$$

Inserting the derived equilibrium expressions for G and nd implies that

$$C_1^2(p, G) = \frac{r(E - C_2^1)}{(1+r)}\left\{\frac{n-1}{n(1-p)} + \frac{1}{n}\right\}.$$

For given n and thus given G the right side is decreasing if p is lowered. The left side, however, is increasing since lowering p essentially gives the first household a better interest rate. Clearly, for p sufficiently close to 1, the right side is greater than the left side, whereas the opposite is true for very small p. Thus, by the mean value theorem, there exists a unique equilibrium.

The equilibrium condition can also be written as

$$C_1^2(p, G) = \frac{r(E - C_2^1)}{(1 + r)} \left\{ \frac{1}{1 - p} - \frac{p}{(1 - p)n} \right\}.$$

I fix an equilibrium value p for some given n. If I increase n, the right side increases, whereas the left side is lowered, since G is lower. To restore the equilibrium condition, by applying my arguments above for the proof of the uniqueness, p has to decline.

In the monopoly case, the second proposition immediately implies that p is equal to one.

Finally, note that $K(n)$ can be written as

$$K = pnd - R = nd(p - 1) + (E - C_2^1 - G).$$

Inserting the equilibrium values for some given n leads to

$$K = -\frac{r(E - C_2^1)(n - 1)}{(1 + r)n} + E - C_2^1 - \frac{r(E - C_2^1)}{(1 + r)n}.$$

This expression is reduced to

$$K = \frac{E - C_2^1}{1 + r}. \qquad \text{Q.E.D.}$$

Proof of Proposition 6. In order to show that free banking is inefficient, I compare the monopoly solution with the limit case $n \to \infty$. Free banking and thus free entry would lead to the limit case since individual profits G_i are positive for every n. G_i corresponds to G/n. Since consumption and, thus, utility of the second generation are the same under free banking and monopoly, I have to compare the utility that the first generation derives from these two cases. The budget constraint of the first generation in the monopoly case is characterized by

$$C_1^1 + C_1^2 = E + G(1)$$

with

$$G(1) = \frac{r(E - C_2^1)}{(1 + r)}.$$

In the case of free banking, the budget constraint amounts to

$$p^{-1}C_1^1 + C_1^2 = p^{-1}E,$$

where p is determined by the equilibrium condition in the time-1 good market for $n \to \infty$ and thus $G(\infty) = 0$. Hence:

$$C_1^2(p, 0) = \frac{r(E - C_2^1)}{(1 + r)(1 - p)}. \tag{A1}$$

Denote this equilibrium value as p^*. In the next step, I show that under the monopoly institution the first generation can afford the optimal consumption plan $C_1^1(p^*, 0)$, $C_1^2(p^*, 0)$ under free banking. Thus, I have to show that

$$C_1^1(p^*, 0) + C_1^2(p^*, 0) \leq E + G(1). \tag{A2}$$

I insert the equilibrium condition for $C_1^2(p^*, 0)$ and the derived expression for $G(1)$ into the budget constraint (A2):

$$C_1^1(p^*, 0) + \frac{r(E - C_2^1)}{(1 + r)} \left\{ \frac{1}{1 - p^*} - 1 \right\} \leq E.$$

This inequality is simply reduced to

$$C_1^1(p^*, 0) + p^* C_1^2(p^*, 0) \leq E$$

which, however, corresponds to the budget constraint under free banking. Thus, in the monopoly case, the first generation can afford just the optimal consumption plan under free banking. However, since the relative price of the time-2 good and the time-1 good under free banking is smaller and different in the monopoly case, the first generation generally attains a higher utility level in the monopoly case. This also means that

$$C_1^1(p^*, 0) < C_1^1(1, G(1)) \qquad \text{and} \qquad C_1^2(p^*, 0) > C_1^2(1, G(1)). \tag{A3}$$

To prove the second assertion, I calculate $R(\infty)$ and $R(1)$ by using the definition of R and the derived equilibrium values for $d(n)$ and $G(n)$. Thus, I obtain

$$R(\infty) = \frac{r(E - C_2^1)}{(1 + r)(1 - p^*)} - (E - C_2^1).$$

From the equilibrium condition (A1), I know that the first term coincides with $C_1^2(p^*, 0)$. Thus

$$R(\infty) = C_1^2(p^*, 0) - (E - C_2^1).$$

On the other hand, I can calculate $R(1)$ as follows:

$$\begin{aligned} R(1) &= d - (E - C_2^1 - G(1)) = E - C_1^1(1, G(1)) - (E - C_2^1 - G(1)) \\ &= C_2^1 + G(1) - C_1^1(1, G(1)). \end{aligned}$$

From the first part of the proof, I already know that $C_1^1(p^*, 0) + C_1^2(p^*, 0) = E + G(1)$. Hence, I can insert $C_1^2(p^*, 0)$ into $R(\infty)$, which leads to

$$R(\infty) = E + G(1) - C_1^1(p^*, 0) - (E - C_2^1) = C_2^1 + G(1) - C_1^1(p^*, 0).$$

Consequently, $R(\infty) > R(1)$ if and only if

$$-C_1^1(p^*, 0) > -C_1^1(1, G(1))$$

which otherwise is true because of (A3). Q.E.D.

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